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ANALYSIS OF A MONOPULSE RADAR.(U)

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ANALYSIS OF A MONOPULSE RADAR

Advanced Simulation Center
US Army Missile Research, Development and Engineering Laboratory
US Army Missile Command
Redstone Arsenal, Alabama 35809

NOVEMBER 1976

Approved for public release; distribution unlimited.

U.S. ARMY MISSILE COMMAND

Redstone Arsenal, Alabama 35809

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A simplified monopulse radar is analyzed and expressions for the outputs of the error detectors are derived with the assumption of no interfering signals being present. Interfering signals are then added to the radar input. A brief look is also taken at a radar antenna control system.

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CONTENTS

I. INTRODUCTION

A simplified monopulse radar will be analyzed in this report. Expressions for the voltages out of the phase detectors of the radar signal processor will be derived with the assumption of no interfering signals being present. Interfering signals due to various sources and their apparent angular position will then be considered. The radar control system will also be analyzed to determine the error present in pointing the antenna when no interfering signals are present.

II. IDEAL MONOPULSE RADAR

A simplified block diagram of a monopulse radar receiver is shown in Figure 1. The sum, azimuth difference, and elevation difference signals are obtained from the hybrid and down converted to an intermediate frequency (IF) where the sum signal is added to the two difference signals. After hard limiting, the sum signal is used as a reference to phase detect the difference channels.

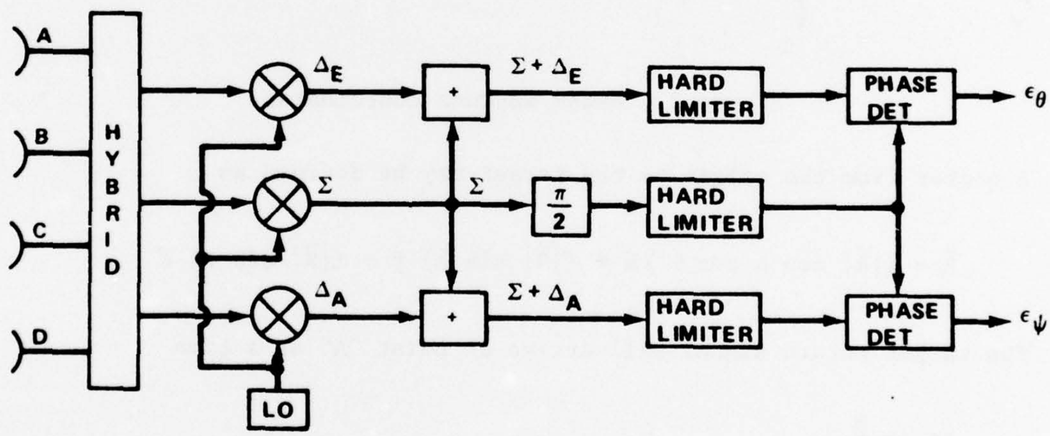

Figure 1. Monopulse radar receiver.

The antenna coordinate system is shown in Figure 2 with the center of the antenna located at the origin. The antenna is assumed to be rectangular in shape with a width of 2 d_1 and a height of 2 d_2. A vector drawn from the origin to a point in quadrant A may be written as

$$\vec{D}_A = 0\,\vec{x} + y\,\vec{y} - z\,\vec{z} \quad .$$

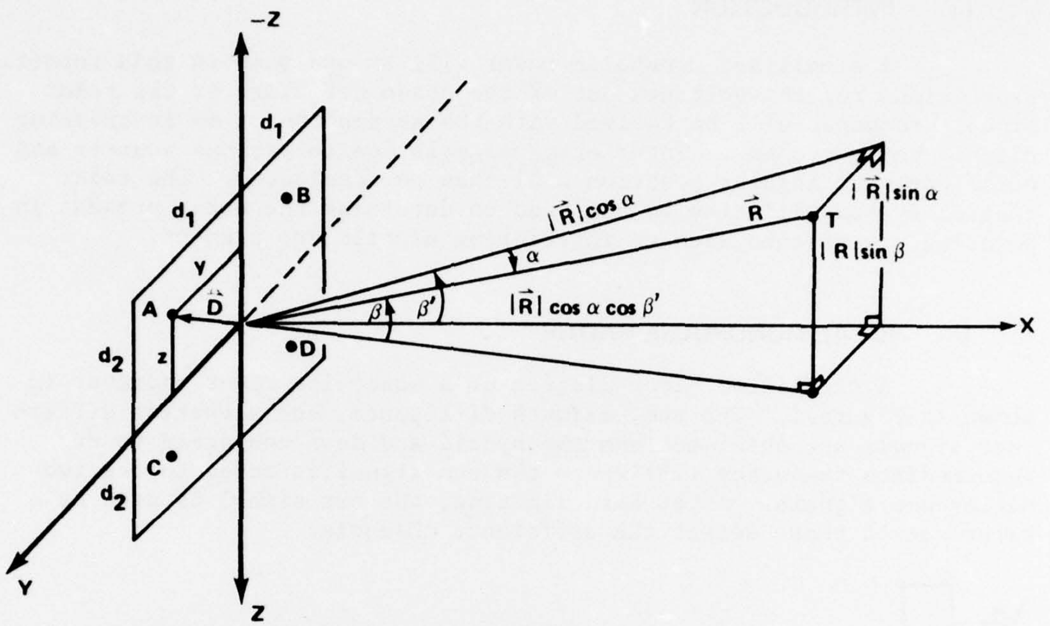

Figure 2. Radar antenna coordinates.

A vector from the origin to the target may be defined as

$$\vec{R} = (|\vec{R}| \cos\alpha \cos\beta')\,\vec{x} + (|\vec{R}| \sin\alpha)\,\vec{y} - (|\vec{R}| \sin\beta)\,\vec{z} \quad .$$

The target return signal will arrive at point "A" at a time

$$\Delta t_A = \frac{S}{C} \quad ,$$

before it arrives at the center of the antenna where S is the projection of vector $\vec{D}_A$ upon vector $\vec{R}$ with an angle γ between vectors $\vec{D}_A$ and $\vec{R}$; i.e.,

$$S = |\vec{D}_A| \cos\gamma \quad .$$

But from the vector dot product

$$|\vec{D}_A| \cos\gamma = \frac{\vec{R} \cdot \vec{D}_A}{|\vec{R}|} \quad .$$

Therefore

$$\Delta t_A = \frac{\vec{R} \cdot \vec{D}_A}{C\ |\vec{R}|} \quad ,$$

or

$$\Delta t_A = \frac{(y \sin \alpha + z \sin \beta)}{C} \quad .$$

This results in a phase difference between point "A" and the center of the antenna of

$$\phi = 2\pi f\ \Delta t_A$$

where f is the frequency of the incoming signal. Since wavelength (λ) is c/f,

$$\phi = \frac{2\pi}{\lambda} (y \sin \alpha + z \sin \beta) \quad .$$

The signals received in each of the quadrants may be obtained from

$$A = K \int_0^{d_1} \int_0^{d_2} e^{j \frac{2\pi}{\lambda} (y \sin \alpha + z \sin \beta)}\ e^{j\omega t}\ d_z\ d_y$$

$$B = K \int_{-d_1}^{0} \int_0^{d_2} e^{j \frac{2\pi}{\lambda} (y \sin \alpha + z \sin \beta)}\ e^{j\omega t}\ d_z\ d_y$$

$$C = K \int_0^{d_1} \int_{-d_2}^{0} e^{j \frac{2\pi}{\lambda} (y \sin \alpha + z \sin \beta)}\ e^{j\omega t}\ d_z\ d_y$$

$$D = K \int_{-d}^{0} \int_{-d_2}^{0} e^{j \frac{2\pi}{\lambda} (y \sin \alpha + z \sin \beta)}\ e^{j\omega t}\ d_z\ d_y \quad .$$

K is the magnitude of the voltage signal appearing at the face of the antenna and $\omega = 2\pi f$. Integrating each of the above equations results in

$$A = K \frac{\left(e^{j \frac{2\pi d_1}{\lambda} \sin \alpha} - 1\right)\left(e^{j \frac{2\pi d_2}{\lambda} \sin \beta} - 1\right)}{\left(j \frac{2\pi}{\lambda} \sin \alpha\right)\left(j \frac{2\pi}{\lambda} \sin \beta\right)} e^{j\omega t}$$

$$B = K \frac{\left(1 - e^{-j \frac{2\pi d_1}{\lambda} \sin \alpha}\right)\left(e^{j \frac{2\pi d_2}{\lambda} \sin \beta} - 1\right)}{\left(j \frac{2\pi}{\lambda} \sin \alpha\right)\left(j \frac{2\pi}{\lambda} \sin \beta\right)} e^{j\omega t}$$

$$C = K \frac{\left(e^{j \frac{2\pi d_1}{\lambda} \sin \alpha} - 1\right)\left(1 - e^{-j \frac{2\pi d_2}{\lambda} \sin \beta}\right)}{\left(j \frac{2\pi}{\lambda} \sin \alpha\right)\left(j \frac{2\pi}{\lambda} \sin \beta\right)} e^{j\omega t}$$

$$D = K \frac{\left(1 - e^{-j \frac{2\pi d_1}{\lambda} \sin \alpha}\right)\left(1 - e^{-j \frac{2\pi d_2}{\lambda} \sin \beta}\right)}{\left(j \frac{2\pi}{\lambda} \sin \alpha\right)\left(j \frac{2\pi}{\lambda} \sin \beta\right)} e^{j\omega t} \quad .$$

The signals out of the hybrid are

$$\Sigma = A + B + C + D$$

$$\Delta_A = (A + C) - (B + D)$$

$$\Delta_E = (A + B) - (C + D) \quad .$$

Performing the indicated operations gives

$$\Sigma = K \frac{\left(e^{j \frac{2\pi d_1}{\lambda} \sin \alpha} - e^{-j \frac{2\pi d_1}{\lambda} \sin \alpha}\right)\left(e^{j \frac{2\pi d_2}{\lambda} \sin \beta} - e^{-j \frac{2\pi d_2}{\lambda} \sin \beta}\right)}{\left(j \frac{2\pi}{\lambda} \sin \alpha\right)\left(j \frac{2\pi}{\lambda} \sin \beta\right)} e^{j\omega t} \quad ,$$

$$\Delta_A = K \frac{\left(e^{j\frac{2\pi d_1}{\lambda}\sin\alpha} + e^{-j\frac{2\pi d_1}{\lambda}\sin\alpha} - 2\right)\left(e^{j\frac{2\pi d_2}{\lambda}\sin\beta} - e^{-j\frac{2\pi d_2}{\lambda}\sin\beta}\right)}{\left(j\,\frac{2\pi}{\lambda}\sin\alpha\right)\left(j\,\frac{2\pi}{\lambda}\sin\beta\right)}\, e^{j\omega t} ,$$

$$\Delta_E = K \frac{\left(e^{j\frac{2\pi d_1}{\lambda}\sin\alpha} - e^{-j\frac{2\pi d_1}{\lambda}\sin\alpha}\right)\left(e^{j\frac{2\pi d_2}{\lambda}\sin\beta} + e^{-j\frac{2\pi d_2}{\lambda}\sin\beta} - 2\right)}{\left(j\,\frac{2\pi}{\lambda}\sin\alpha\right)\left(j\,\frac{2\pi}{\lambda}\sin\beta\right)}\, e^{j\omega t} .$$

The following trigonometric identities will now be used:

$$e^{jx} - e^{-jx} = j2 \sin x$$

$$e^{jx} + e^{-jx} = 2 \cos x \quad .$$

Thus the signals out of the hybrid may be written as

$$\Sigma = |\Sigma|\, e^{j\omega t}$$

$$\Delta_A = j\, |\Delta_A|\, e^{j\omega t}$$

$$\Delta_E = j\, |\Delta_E|\, e^{j\omega t} \quad ,$$

where

$$|\Sigma| = 4K\, d_1\, d_2 \left(\frac{\sin\left(\frac{2\pi d_1}{\lambda}\sin\alpha\right)}{\frac{2\pi d_1}{\lambda}\sin\alpha}\right)\cdot\left(\frac{\sin\left(\frac{2\pi d_2}{\lambda}\sin\beta\right)}{\frac{2\pi d_2}{\lambda}\sin\beta}\right) \tag{1}$$

$$|\Delta_A| = 4K\, d_1\, d_2 \left(\frac{1 - \cos\left(\frac{2\pi d_1}{\lambda}\sin\alpha\right)}{\frac{2\pi d_1}{\lambda}\sin\alpha}\right)\cdot\left(\frac{\sin\left(\frac{2\pi d_2}{\lambda}\sin\beta\right)}{\frac{2\pi d_2}{\lambda}\sin\beta}\right) \tag{2}$$

$$|\Delta_E| = 4K\, d_1\, d_2 \left(\frac{\sin\left(\frac{2\pi d_1}{\lambda} \sin\alpha\right)}{\frac{2\pi d_1}{\lambda} \sin\alpha} \right) \cdot \left(\frac{1 - \cos\left(\frac{2\pi d_2}{\lambda} \sin\beta\right)}{\frac{2\pi d_2}{\lambda} \sin\beta} \right) \quad . \tag{3}$$

The three signals are mixed to an IF where the Σ signal is added to the Δ signals and the resulting signals are hard limited to a magnitude K_L.

$$\Sigma + \Delta_A = K_L\, e^{j\phi_1}\, e^{j\omega_{IF} t}$$

$$\Sigma + \Delta_E = K_L\, e^{j\phi_2}\, e^{j\omega_{IF} t} \quad .$$

Phase angles ϕ_1 and ϕ_2 are defined by

$$\phi_1 = \sin^{-1}\left(\frac{|\Delta_A|}{\sqrt{|\Sigma|^2 + |\Delta_A|^2}} \right) \tag{4}$$

$$\phi_2 = \sin^{-1}\left(\frac{|\Delta_E|}{\sqrt{|\Sigma|^2 + |\Delta_E|^2}} \right) \quad . \tag{5}$$

Substituting Equations (1), (2), and (3) into Equations (4) and (5) gives

$$\phi_1 = \sin^{-1}\left(\frac{1 - \cos\left(\frac{2\pi d_1}{\lambda} \sin\alpha\right)}{\sqrt{\sin^2\left(\frac{2\pi d_1}{\lambda} \sin\alpha\right) + \left[1 - \cos\left(\frac{2\pi d_1}{\lambda} \sin\alpha\right)\right]^2}} \right)$$

$$\phi_1 = \sin^{-1}\left(\frac{1 - \cos\left(\frac{2\pi d_1}{\lambda} \sin\alpha\right)}{\sqrt{2\left[1 - \cos\left(\frac{2\pi d_1}{\lambda} \sin\alpha\right)\right]}} \right)$$

$$\phi_1 = \frac{\pi d_1}{\lambda} \sin\alpha \quad . \tag{6}$$

Similarly ϕ_2 is

$$\phi_2 = \frac{\pi d_2}{\lambda} \sin \beta \quad . \tag{7}$$

The Σ signal is shifted in phase by $\pi/2$ and is also hard limited giving

$$\Sigma = j K_L e^{j\omega_{IF} t} \quad .$$

Since only the real parts of the signals are of interest, the three signals may be written as

$$\Sigma = -K_L \sin (\omega_{IF} t)$$

$$\Sigma + \Delta_A = K_L \cos (\omega_{IF} t + \phi_1)$$

$$\Sigma + \Delta_E = K_L \cos (\omega_{IF} t + \phi_2) \quad .$$

The Σ signal is used as the reference signal for the phase detectors. The output of the azimuth channel phase detector is

$$\varepsilon_\psi = [-K_L \sin (\omega_{IF} t)] \, [K_L \cos (\omega_{IF} t + \phi_1)] \quad .$$

Using only the lower sideband,

$$\varepsilon_\psi = K_\psi \sin \phi_1 \quad .$$

But from Equation (4) it can be seen that

$$\varepsilon_\psi = K_\psi \frac{|\Delta_A|}{\sqrt{|\Sigma|^2 + |\Delta_A|^2}} \quad . \tag{8}$$

Similarly the elevation channel phase detector output, ε_θ, is

$$\varepsilon_\theta = K_\theta \frac{|\Delta_E|}{\sqrt{|\Sigma|^2 + |\Delta_E|^2}} \quad . \tag{9}$$

K_ψ and K_θ are peak magnitudes of the error signals. Using Equations (6) and (7), the outputs of the phase detectors may be written as

$$\varepsilon_\psi = K_\psi \sin\left(\frac{\pi d_1}{\lambda} \sin\alpha\right) \tag{10}$$

$$\varepsilon_\theta = K_\theta \sin\left(\frac{\pi d_2}{\lambda} \sin\beta\right) \quad . \tag{11}$$

Since the error signals as given in Equations (10) and (11) are functions of $\sin\alpha$ and $\sin\beta$, these two quantities will now be defined in terms of the antenna azimuth and elevation angles, and the target azimuth and elevation angles as shown in Figure 3.

From Figure 3 it can be seen that:

$$\sin\alpha = \cos\theta_B \sin(\psi_B - \psi_A) \quad . \tag{12}$$

The determination of $\sin\beta$ is more complicated, but can be determined as follows:

$$\cos\alpha = \sqrt{1 - \cos^2\theta_B \sin^2(\psi_B - \psi_A)}$$

$$\sin\beta' = \frac{\sin\beta}{\cos\alpha}$$

$$\sin\theta_B' = \frac{\sin\theta_B}{\cos\alpha}$$

$$\cos\beta' = \sqrt{\frac{\cos^2\alpha - \sin^2\beta}{\cos^2\alpha}}$$

$$\cos\theta_B' = \frac{\cos\theta_B \cos(\psi_B - \psi_A)}{\cos\alpha}$$

$$\sin\theta_A = \sin(\theta_B' - \beta') \quad .$$

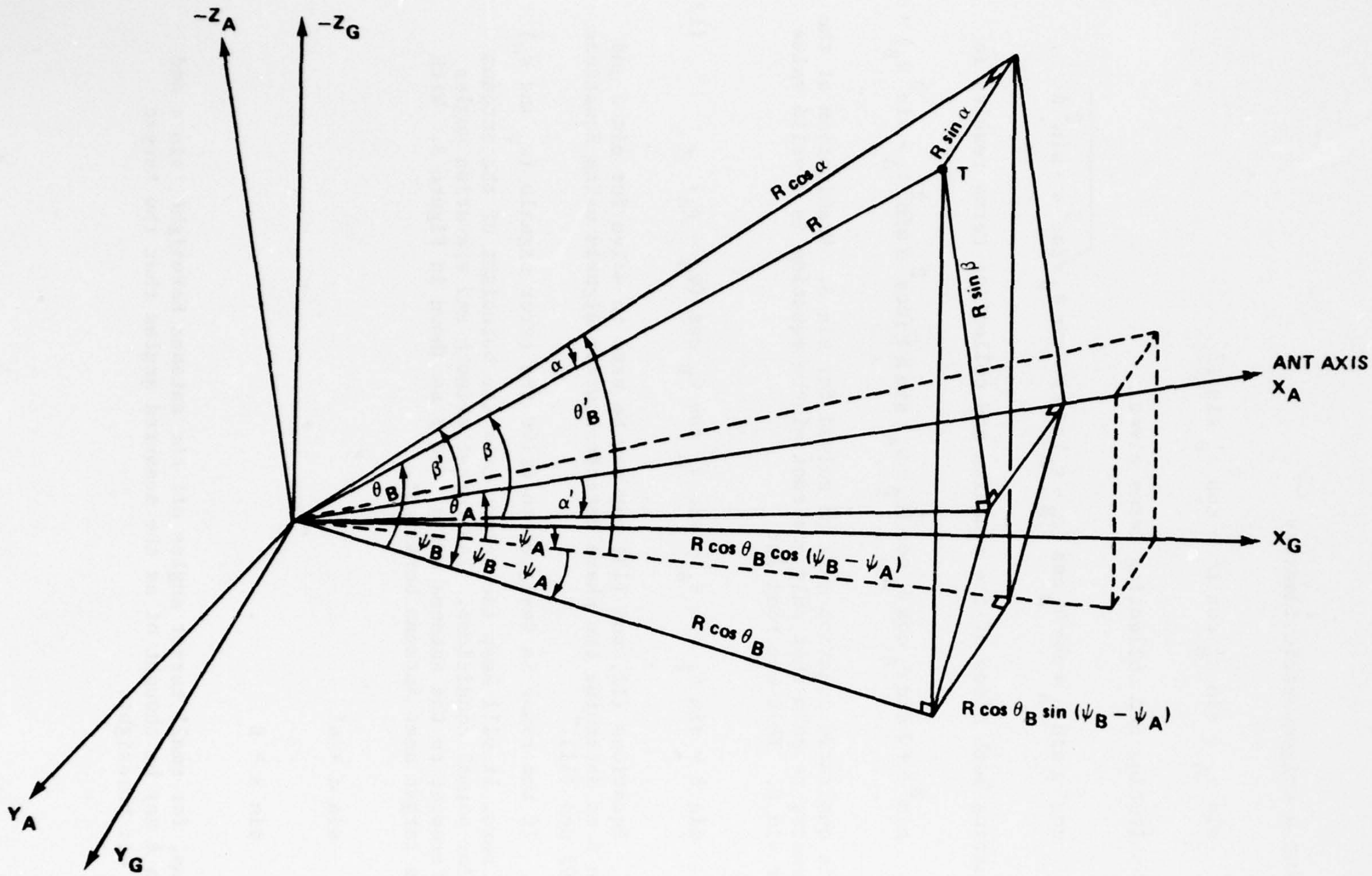

Figure 3. Antenna and ground coordinate systems.

Using a trigonometric identity

$$\sin \theta_A = \sin \theta'_B \cos \beta' - \cos \theta'_B \sin \beta' \quad .$$

Substituting and collecting terms gives

$$\cos^2 \alpha \sin \theta_A + \cos \theta_B \cos (\psi_B - \psi_A) \sin \beta = \sin \theta_B \sqrt{\cos^2 \alpha - \sin^2 \beta} \quad .$$

Squaring both sides of the equation and collecting terms results in

$$\sin^2 \beta + 2 \sin \theta_A \cos \theta_B \cos (\psi_B - \psi_A) \sin \beta + (\cos^2 \alpha \sin^2 \theta_A - \sin^2 \theta_B) = 0 \quad .$$

This quadratic equation must be solved for $\sin \beta$. Examination of the geometry reveals that only one root of the equation is a valid value for $\sin \beta$. This one root is:

$$\sin \beta = \sin \theta_B \cos \theta_A - \sin \theta_A \cos \theta_B \cos (\psi_B - \psi_A) \quad . \qquad (13)$$

Equations (12) and (13) may now be used to solve for $\sin \alpha$ and $\sin \beta$ to determine the phase detectors error signals using Equations (10) and (11).

If the radar is designed to drive the error signals (ε_ψ and ε_θ) to zero, it will keep the target near the boresight of the antenna under normal conditions. The target azimuth and elevation angles referenced to the antenna are α' and β as shown in Figure 3. With the target near antenna boresight,

$$\sin \alpha \approx \alpha'$$

$$\sin \beta \approx \beta \quad .$$

Thus, for small target angles off the antenna boresight, $\sin \alpha$ and $\sin \beta$ may be thought of as the measured angles that the target is off boresight.

Examination of Equations (1), (2), and (3) reveals that if the target is near antenna boresight in one axis, then the Σ and Δ signals for the other channel are approximated by

$$|\Sigma|_P = \frac{\sin\left(\frac{2\pi d}{\lambda}\sin\theta_T\right)}{\frac{2\pi d}{\lambda}\sin\theta_T} \tag{14}$$

$$|\Delta|_P = \frac{1-\cos\left(\frac{2\pi d}{\lambda}\sin\theta_T\right)}{\frac{2\pi d}{\lambda}\sin\theta_T} \quad . \tag{15}$$

$|\Sigma|_P$ and $|\Delta|_P$ are the voltage antenna patterns for the Σ and Δ channels. θ_T is used to denote target angle off boresight for either the azimuth or elevation channel and the subscript of d is dropped so that d denotes the dimension in either channel.

The 3 dB beamwidth (BW) of the Σ pattern occurs when

$$\frac{\sin\left[\frac{2\pi d}{\lambda}\sin\left(\frac{BW}{2}\right)\right]}{\frac{2\pi d}{\lambda}\sin\frac{BW}{2}} = \frac{\sqrt{2}}{2} \quad .$$

With a small beamwidth

$$\sin\left(\pi\frac{d}{\lambda}BW\right) = \frac{\sqrt{2}}{2}\left(\pi\frac{d}{\lambda}BW\right) \quad .$$

Solving for d/λ in terms of beamwidth results in

$$\frac{d}{\lambda} = \frac{0.443}{BW} \quad . \tag{16}$$

Substituting this value of d/λ into Equations (14) and (15) and using the small angle approximation for $\sin\theta_T$ gives

$$|\Sigma|_P = \frac{\sin\left(0.886\,\pi\frac{\theta_T}{BW}\right)}{\left(0.886\,\pi\frac{\theta_T}{BW}\right)} \tag{17}$$

$$|\Delta|_P = \frac{1 - \cos\left(0.886\ \pi\ \frac{\theta_T}{BW}\right)}{\left(0.886\ \pi\ \frac{\theta_T}{BW}\right)} \quad . \tag{18}$$

Plots of 20 log $|\Sigma|_P$ and 20 log $|\Delta|_P$ are shown in Figure 4 as a function of θ_T/BW.

III. INTERFERENCE

It has been shown that the general equations for Σ and Δ signals may be written as

$$\Sigma = \Sigma_T\ e^{j\omega_{IF}t}$$

$$\Delta = j\ \Delta_T\ e^{j\omega_{IF}t}$$

where Σ_T and Δ_T are the magnitudes of the target signal in the Σ and Δ channels. If interference signals are present (e.g., thermal noise which is always present, or clutter) the true target signal is corrupted and the Σ and Δ signals may then be written as

$$\Sigma = \left[\left(\Sigma_T + \sum_{i=1}^{n} \Sigma_{iI}\right) + j \sum_{i=1}^{n} \Sigma_{iQ}\right] e^{j\omega_{IF}t}$$

$$\Delta = j \left[\left(\Delta_T + \sum_{i=1}^{n} \Delta_{iI}\right) + j \sum_{i=1}^{n} \Delta_{iQ}\right] e^{j\omega_{IF}t}$$

where the subscript "i" denotes interference signal "i" out of a total of "n" interference signals. The subscript "I" denotes being in phase with the target signal and the subscript "Q" denotes being in quadrature with the target signal. The $\Sigma + \Delta$ signal is formed and limited to obtain

$$\Sigma + \Delta = K_L\ e^{j\phi_\Delta}\ e^{j\omega_{IF}t}$$

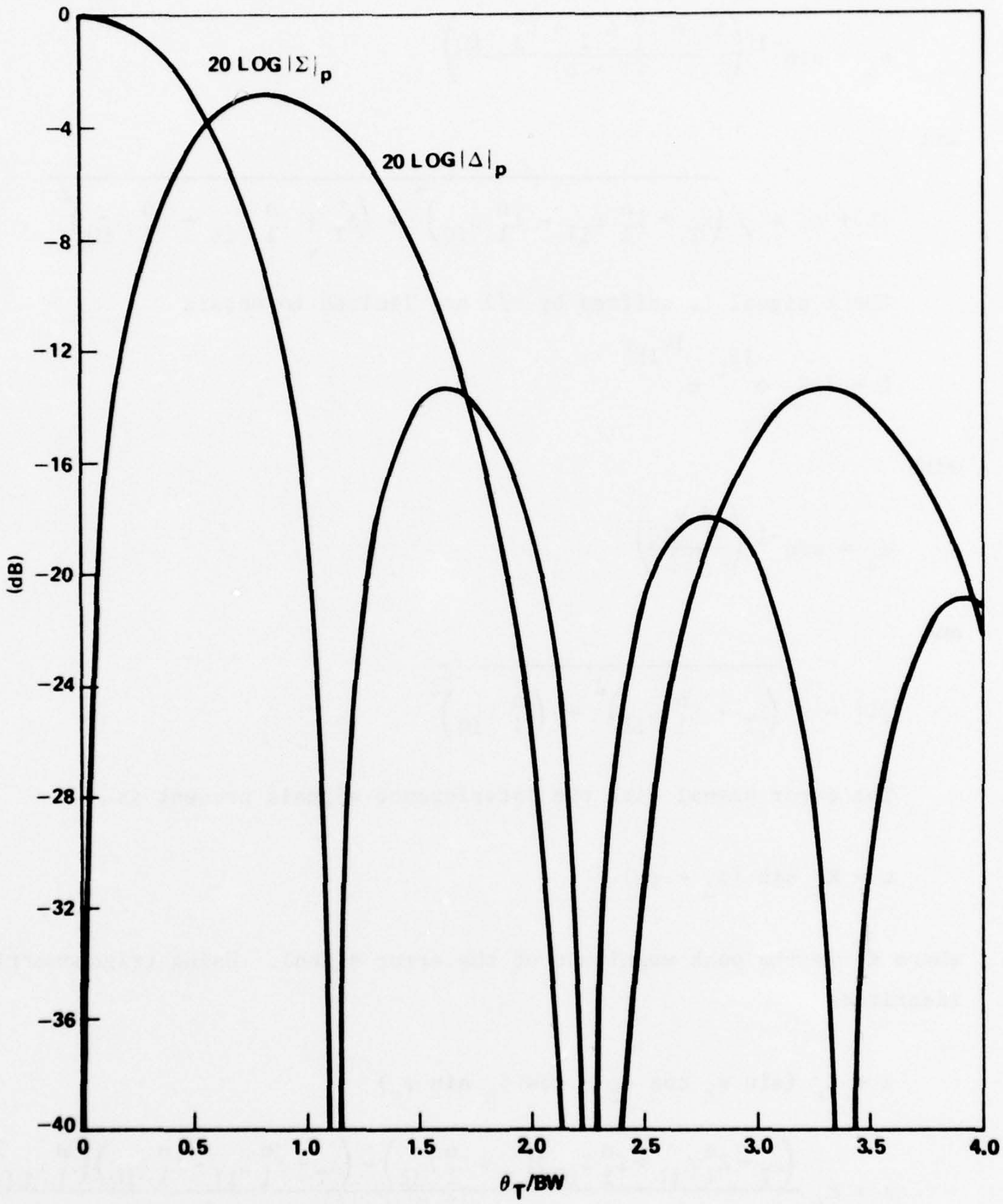

Figure 4. Sum and difference antenna patterns.

with

$$\phi_\Delta = \sin^{-1}\left(\frac{\Delta_T + \Sigma_1^n \Delta_{iI} + \Sigma_1^n \Sigma_{iQ}}{|\Sigma + \Delta|}\right)$$

and

$$|\Sigma + \Delta| = \sqrt{\left(\Sigma_T + \Sigma_1^n \Sigma_{iI} - \Sigma_1^n \Delta_{iQ}\right)^2 + \left(\Delta_T + \Sigma_1^n \Delta_{iI} + \Sigma_1^n \Sigma_{iQ}\right)^2} \; .$$

The Σ signal is shifted by $\pi/2$ and limited to obtain

$$\Sigma = J\, K_L\, e^{j\phi_\Sigma}\, e^{j\omega_{IF} t}$$

with

$$\phi_\Sigma = \sin^{-1}\left(\frac{\Sigma_1^n \Sigma_{iQ}}{|\Sigma|}\right)$$

and

$$|\Sigma| = \sqrt{\left(\Sigma_T + \Sigma_1^n \Sigma_{iI}\right)^2 + \left(\Sigma_1^n \Sigma_{iQ}\right)^2} \; .$$

The error signal with the interference signals present is

$$\varepsilon = K_\varepsilon \sin\left(\phi_\Delta - \phi_\Sigma\right)$$

where K_ε is the peak magnitude of the error signal. Using trigonometric identities

$$\varepsilon = K_\varepsilon \left(\sin \phi_\Delta \cos \phi_\Sigma - \cos \phi_\Delta \sin \phi_\Sigma\right)$$

$$\varepsilon = K_\varepsilon \frac{\left(\Delta_T + \Sigma_1^n \Delta_{iI} + \Sigma_1^n \Sigma_{iQ}\right)\left(\Sigma_T + \Sigma_1^n \Sigma_{iI}\right) - \left(\Sigma_T + \Sigma_1^n \Sigma_{iI} - \Sigma_1^n \Delta_{iQ}\right)\left(\Sigma_1^n \Sigma_{iQ}\right)}{|\Sigma + \Delta|\; |\Sigma|} \; .$$

$$\varepsilon = K_\varepsilon \frac{\left(\Sigma_T + \Sigma_1^n \Sigma_{iI}\right)\left(\Delta_T + \Sigma_1^n \Delta_{iI}\right) + \left(\Sigma_1^n \Delta_{iQ}\right)\left(\Sigma_1^n \Sigma_{iQ}\right)}{|\Sigma + \Delta|\; |\Sigma|} \; . \qquad (19)$$

If it is assumed that the Σ channel target signal is much greater than the Δ channel target signal and all the interference signals, ε may be approximated by

$$\varepsilon = K_\varepsilon \frac{\Sigma_T \left(\Delta_T + \Sigma_1^n \Delta_{iI}\right)}{\Sigma_T^2} ,$$

or

$$\varepsilon = K_\varepsilon \frac{\Delta_T + \Sigma_1^n \Delta_{iI}}{\Sigma_T} . \qquad (20)$$

With a null tracking system, the servo tries to make $\varepsilon = 0$. Therefore,

$$\frac{\Delta_T}{\Sigma_T} = - \frac{\Sigma_1^n \Delta_{iI}}{\Sigma_T} .$$

The Σ channel target signal power is

$$S_\Sigma = \Sigma_T^2 .$$

The interference power in the Δ channel is

$$\Sigma_1^n I_{\Delta i} = \Sigma_1^n \left(\Delta_{iI}^2 + \Delta_{iQ}^2\right) .$$

If it is assumed that the average value of Δ_{iQ} is equal to the average value of Δ_{iI}, then

$$\Delta_{iI} = \sqrt{\frac{I_{\Delta i}}{2}} ,$$

and

$$\Sigma_1^n \Delta_{iI} = \Sigma_1^n \sqrt{\frac{I_{\Delta i}}{2}} \quad .$$

Then

$$\frac{\Delta_T}{\Sigma_T} = - \frac{\Sigma_1^n \sqrt{\frac{I_{\Delta i}}{2}}}{\sqrt{S_\Sigma}}$$

or

$$\frac{\Delta_T}{\Sigma_T} = - \Sigma_1^n \frac{1}{\sqrt{2\left(\frac{S_\Sigma}{I_{\Delta i}}\right)}} \quad . \qquad (21)$$

If $\Sigma_T \gg \Delta_T$, the phase detector error signal as given by Equations (8) and (9) reduces to

$$\varepsilon = \frac{\Delta_T}{\Sigma_T} \quad ,$$

and substituting into Equations (10) and (11)

$$\frac{\Delta_T}{\Sigma_T} = K_\varepsilon \sin\left(\frac{\pi d}{\lambda} \sin \theta_T\right) \quad .$$

Expressing d/λ in terms of BW as given in Equation (16), setting $K_\varepsilon = 1$, and assuming that $\pi d/\lambda \sin \theta_T$ is small gives

$$\frac{\Delta_T}{\Sigma_T} = \pi \frac{0.443}{BW} \theta_T \quad .$$

Substituting into Equation (21), gives

$$0.443\, \pi \frac{\theta_T}{BW} = -\Sigma_1^n \frac{\sqrt{I_{\Delta i}}}{\sqrt{2\, S_\Sigma}} \quad .$$

Solving for θ_T will give the angle at which the target signal must appear to null the interference. The negative of this value will give the angular location of the composite interference signal. This interference angle θ_I is

$$\theta_I = \frac{BW}{k_m} \Sigma_1^n \sqrt{\frac{I_{\Delta i}}{2\ S_\Sigma}}$$

where

$$k_m = 0.443\ \pi \approx 1.4 \quad .$$

The interference power $I_{\Delta i}$ must be divided by the number of independent samples of interference (n_{ei}) that occur over the measurement interval. Thus

$$\theta_I = \frac{BW}{k_m} \Sigma_1^n \sqrt{\frac{I_{\Delta i}}{2\ S_\Sigma\ n_{ei}}} \quad . \tag{22}$$

If the interference signals are Gaussian with a 0 mean, the 1 σ value of interference is

$$\sigma = \frac{BW}{k_m} \sqrt{\Sigma_1^n \frac{I_{\Delta i}}{2\ S_\Sigma\ n_{ei}}} \quad .$$

Defining σ_i as

$$\sigma_i = \frac{BW}{k_m \sqrt{2\left(\frac{S_\Sigma}{I_{\Delta i}}\right) n_{ei}}} \tag{23}$$

results in

$$\sigma^2 = \sigma_1^2 + \sigma_2^2 + \ldots + \sigma_n^2 \quad . \tag{24}$$

Solving Equation (23) for $\sqrt{I_{\Delta i}/n_{ei}}$ gives the interference voltage signal produced by the 1 σ angular error σ_i as

$$\sqrt{\frac{I_{\Delta i}}{n_{ei}}} = \frac{k_m}{BW} \sqrt{2\ S_\Sigma}\ \sigma_i \quad .$$

For modeling purposes, the 1 σ values are multiplied by a Gaussian random number having a mean of 0 and a standard deviation of 1. Defining this random number as RN_i results in an interference voltage sample of

$$\sqrt{\frac{I_{\Delta i}}{n_{ei}}} = \frac{k_m}{BW} \sqrt{2\, S_\Sigma}\ \sigma_i\ RN_i \quad .$$

Substituting into Equation (22), the total interference signal becomes

$$\theta_I = \sigma_1\ RN_1 + \sigma_2\ RN_2 + \ldots + \sigma_n\ RN_n \quad . \tag{25}$$

Equation (25) gives the error signal due to multiple interference signals in terms of the errors caused by the individual error sources. However, it must be remembered that this equation was derived with the assumption of small angles and the assumption that the Σ channel target signal is much greater than any of the other signals. If these assumptions do not hold in any given circumstance, then Equation (25) may not be valid. Finally, it should be noted that the signal-to-interference ratio is the ratio of the Σ channel target signal power to the Δ channel interference power.

A. Thermal Noise

The 1 σ value of thermal noise may be obtained by using Equation (23). If the noise levels are the same in each channel then

$$\frac{S_\Sigma}{I_{\Delta i}} = \frac{S_\Sigma}{N_\Sigma} \quad .$$

Dropping the Σ subscript, the 1 σ value of thermal noise is

$$\sigma_N = \frac{BW}{k_m \sqrt{2 \left(\frac{S}{N}\right) n_{en}}} \tag{26}$$

where

$$\frac{S}{N} = \frac{P_T\ G^2\ \lambda^2\ \sigma_T}{(4\pi)^3\ K\ T\ B\ \overline{NF}\ L\ R^4} \tag{27}$$

P_T = Peak transmitter power

G = Antenna gain

λ = Operating wavelength

σ_T = Radar cross-section of target

K = Boltzmann's constant (1.38×10^{-23} J/°)

T = Absolute temperature (usually taken as 290°K)

B = Receiver bandwidth

$\overline{NF}$ = Receiver noise figure

L = Radar losses

R = Radar range to target

$$n_{en} = \frac{PRF}{2 \times \beta_n} \tag{28}$$

where β_n is the servo noise bandwidth. Defining k_n as

$$k_n = \frac{P_T\ G^2\ \lambda^2\ \sigma_T}{(4\pi)^3\ K\ T\ B\ \overline{NF}\ L} \tag{29}$$

then

$$\sigma_N = \frac{BW}{k_m\ \sqrt{2\ k_n\ n_{en}}}\ R^2 \quad .$$

Defining

$$VT1 = \frac{BW}{k_m\ \sqrt{2\ k_n\ n_{en}}} \tag{30}$$

gives

$$\sigma_N = VT1 \times R^2 \quad . \tag{31}$$

B. Rain Interference

The presence of rain will both attenuate the target signal and create a signal return from the rain itself. The attenuation effect will be discussed first. If the amount of signal attenuation

is defined as A_{TTEN}^2, then the 1 σ error due to thermal noise in rain is

$$\sigma_N = VT1 \times R^2 \times A_{TTEN} \quad . \tag{32}$$

The amount of attenuation present may be calculated from

$$A_{dB} = A_{TT} \times R$$

where A_{dB} is the total rain attenuation in dB, A_{TT} is the two-way rain attenuation expressed in dB/meter, and R is the range in meters. Then the total rain attenuation is

$$A = 10^{(A_{dB}/10)}$$

or

$$A = 10^{\left(\frac{ATT \times R}{10}\right)} \quad .$$

Since $A = A_{TTEN}^2$

$$A_{TTEN} = 10^{\left(\frac{ATT \times R}{20}\right)} \quad . \tag{33}$$

The 1 σ value for rain return may be calculated from

$$\sigma_R = \frac{BW}{k_m \sqrt{2\left(\frac{S_\Sigma}{I_{R\Delta}}\right) n_{er}}} \quad . \tag{34}$$

If the rain is uniformly distributed over all regions in which a significant response would be obtained

$$\frac{S_\Sigma}{I_{R\Delta}} = \frac{S}{I_R}$$

where S/I_R is the sum channel signal-to-rain interference ratio. The volume of rain illuminated can be obtained from the dimensions of the resolution cell and is

$$\text{Volume} = (R \times BW_{el})(R \times BW_{az}) \frac{c\tau}{2}$$

or

$$\text{Volume} = R^2\ BW_{el}\ BW_{az} \frac{c\tau}{2}$$

where

BW_{el}, BW_{az} = 3 dB antenna BW in azimuth and elevation

τ = Radar pulse width.

The effective rain cross-section is

$$\sigma_{rain} = \frac{\text{Volume} \times \eta_v}{L_p^2\ L_{cp}} \qquad (35)$$

where

L_p = Two way beamshape loss (L_p^2 = loss in both coordinates)

L_{cp} = Loss to rain return if circular polarization is used

η_v = Volume reflectivity of rain (m^2/m^3).

L_p is 1.329 for a radar with uniform illumination*. Thus, L_p^2 = 1.77. η_v may be obtained from curves in several different textbooks or it may be calculated using the following equation**

$$\eta_v = 6 \times 10^{-14}\ r^{1.6}\ \lambda^{-4} \qquad (36)$$

*Barton and Ward, Handbook of Radar Measurement, New York: Prentice Hall, Inc., 1969, p. 34 (Table 2.4).
**Ibid., p. 138.

where r is the rainfall rate in mm/hr. A loss due to circular polarization would probably be between 15 and 20 dB. The Σ channel S/I_R is then

$$\frac{S}{I_R} = \frac{1.77\ \sigma_T\ L_{cp}}{\eta_v\ BW_{el}\ BW_{az}\ \frac{c\tau}{2}\ R^2} \quad . \tag{37}$$

The number of independent samples of rain return may be calculated from

$$n_{er} = \frac{1}{t_R \times 2\ \beta_n} \tag{38}$$

where

$$t_R = \frac{1}{\sigma_{DR}}$$

$$\sigma_{DR} = \frac{2\ \sigma_{VR}}{\lambda}$$

t_R = Correlation time for rain

σ_{DR} = 1 σ doppler spread on rain

σ_{VR} = 1 σ spread on rain velocity due to wind.

σ_{VR} has a value between 2 and 4 m/sec.*

Defining

$$K_R = \frac{1.77\ \sigma_T\ L_{cp}}{\eta_v\ BW_{el}\ BW_{az}\ \frac{c\tau}{2}} \tag{39}$$

---

*Ibid., p. 139.

gives

$$\sigma_R = \frac{BW}{k_m \sqrt{2\left(\frac{K_R}{R^2}\right) n_{er}}} \quad .$$

Defining

$$VEP = \frac{BW}{k_m \sqrt{2\ K_R\ n_{er}}} \tag{40}$$

gives

$$\sigma_R = VEP \times R \quad . \tag{41}$$

C. Ground Clutter

The 1 σ value for ground clutter return may be calculated from

$$\sigma_c = \frac{BW}{k_m \sqrt{2\left(\frac{S_\Sigma}{C_\Delta}\right) n_{ec}}} \quad . \tag{42}$$

where

C_Δ = Clutter power in Δ channel

$$n_{ec} = \frac{1}{t_c \times 2\ \beta_n} \tag{43}$$

$$t_c = \frac{1}{\sigma_{DC}}$$

$$\sigma_{DC} = \frac{2\ \sigma_{vc}}{\lambda}$$

t_c = Correlation time for clutter

σ_{DC} = 1 σ doppler spread on clutter

σ_{vc} = 1 σ spread on clutter velocity due to wind.

σ_{vc} has a value between $\approx$ 0.0 and 0.4 m/sec depending upon the type of clutter and the wind velocity.* If the clutter is uncorrelated from pulse to pulse, it then appears similar to thermal noise and the correlation time is the reciprocal of the PRF.

The effective cross-section for Σ channel clutter that is uniformly distributed in range and azimuth and has a Rayleigh amplitude distribution is

$$\sigma_{cc} = \sigma^0 G_{SC}^2 \frac{R \times BW_{az}}{L_p} \frac{c\tau}{2} \tag{44}$$

where

σ^0 = Clutter reflectivity (m^2/m^2)

G_{SC} = Σ channel antenna gain to clutter.

The clutter reflectivity can vary from at least -10 dB to -30 dB below 1 m^2 depending upon the type of clutter, the angle of incidence, and the wavelength. Typical values may be between -15 dB and -25 dB.

The Σ channel signal-to-clutter ratio is

$$\frac{S}{C} = \frac{G_{ST}^2}{G_{SC}^2} \frac{\sigma_T}{\sigma^0} \frac{L_p}{R \times BW_{az}} \frac{2}{c\tau} I_D \tag{45}$$

where

G_{ST} = Σ channel antenna gain to target signal

I_D = Doppler improvement factor in signal-to-clutter power ratio.

Defining

$$G_{SS} = \frac{G_{ST}}{G_{SC}} \tag{46}$$

*Ibid., p. 139.

and

$$K_c = \frac{\sigma_T}{\sigma_0} \frac{L_p}{BW_{az}} \frac{2}{c\tau} I_D \tag{47}$$

gives

$$\frac{S}{C} = \frac{G_{SS}^2 K_c}{R} \quad . \tag{48}$$

If the clutter is distributed over several beamwidths in azimuth around the target, the clutter power in the azimuth Δ channel is equal to the clutter power in the Σ channel because the channels are adjusted for equal noise outputs and the 1 σ value for clutter in the azimuth channel is

$$\sigma_{CA} = \frac{BW_{az}}{k_m \sqrt{2K_c n_{ec}}} \frac{\sqrt{R}}{G_{SS}} \quad . \tag{49}$$

Defining

$$VRG = \frac{BW_{az}}{k_m \sqrt{2 K_c n_{ec}}} \tag{50}$$

gives

$$\sigma_{CA} = VRG \frac{\sqrt{R}}{G_{SS}} \quad . \tag{51}$$

In the elevation channel, clutter is not uniformly distributed over several beamwidths and most of the clutter returns come from an elevation angle θ_B below the target. Under these conditions the clutter power in the elevation Δ channel (C_Δ) is not equal to the clutter power in the Σ channel. The transmit antenna gain to the clutter will be G_{SC}, but the elevation Δ channel antenna gain to the clutter will be G_{DC}. Then

$$\frac{S}{C_\Delta} = \frac{G_{ST}^2}{G_{SC}\, G_{DC}} \frac{\sigma_T}{\sigma_0} \frac{L_p}{R \times BW_{az}} \frac{2}{c\tau} I_D \quad .$$

Defining

$$G_{DS} = \frac{G_{ST}}{G_{DC}} \tag{52}$$

gives

$$\frac{S}{C_\Delta} = \frac{G_{SS}\ G_{DS}\ K_c}{R} \quad . \tag{53}$$

Substituting Equation (53) into Equation (41) gives

$$\sigma_{CE} = \frac{BW_{el}}{k_m \sqrt{2\ K_c\ n_{ec}}} \sqrt{\frac{R}{G_{SS}\ G_{DS}}} \quad . \tag{54}$$

Defining

$$VRS = \frac{BW_{el}}{k_m \sqrt{2\ K_c\ n_{ec}}} \tag{55}$$

gives

$$\sigma_{CE} = VRS \sqrt{\frac{R}{G_{SS}\ G_{DS}}} \quad . \tag{56}$$

D. Multipath

The 1 σ value for error due to multipath in the elevation channel may be calculated from

$$\sigma_M = \frac{BW_{el}}{k_m \sqrt{2\left(\frac{S_\Sigma}{I_M}\right) n_{em}}} \tag{57}$$

where

I_M = Δ channel interference power due to multipath.

Then

$$\frac{S_\Sigma}{I_M} = \frac{G_{ST}}{\rho^2\ G_{DM}} = \frac{G_{DS}}{\rho^2} \tag{58}$$

where

G_{DM} = Δ channel antenna gain to multipath

ρ = Surface reflection coefficient (voltage).

This assumes that the target is illuminated by the full gain of the Σ channel and that equal power is scattered along the direct and indirect paths. A flat earth with a single reflection point is assumed and this places the multipath at an angle θ_M below the target as shown in Figure 5.

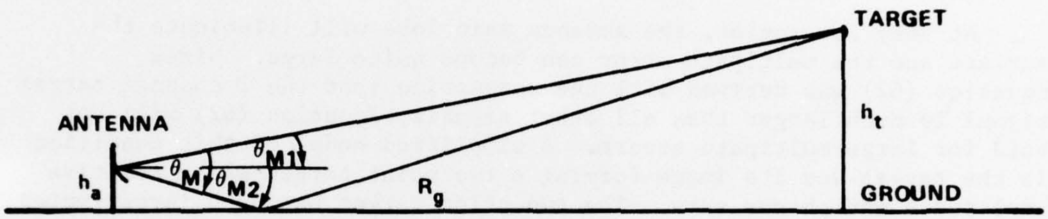

Figure 5. Geometry of multipath model.

$$\theta_M = \theta_{M1} + \theta_{M2} \tag{59}$$

$$\theta_{M1} = \tan^{-1}\left(\frac{h_t - h_a}{R_g}\right)$$

$$\theta_{M2} = \tan^{-1}\left(\frac{h_t + h_a}{R_g}\right)$$

h_t = Target height

h_a = Antenna height

R_g = Target ground range.

The Δ channel antenna gain to the multipath is then the Δ channel gain at angle θ_M.

Since filtering is usually ineffective for multipath, n_{em} can be assumed to be 1, and the 1 σ error becomes

$$\sigma_M = \frac{\rho \times BW_{el}}{k_m \sqrt{2}\, G_{DS}} \quad . \tag{60}$$

Defining

$$VMP = \frac{\rho \times BW_{el}}{k_m \sqrt{2}} \qquad (61)$$

gives

$$\sigma_M = VMP \frac{1}{\sqrt{G_{DS}}} \quad . \qquad (62)$$

At very low angles, the antenna main lobe will illuminate the surface and the multipath error can become quite large. Since Equation (62) was derived with the assumption that the Σ channel target signal is much larger than all other signals, Equation (62) will not hold for large multipath errors. A simplified model of this condition is the target and its image forming a two point target whose relative amplitudes and phases vary. The two point target has been investigated in several textbooks and will not be discussed further except to state that it can be shown that the apparent radar center of the two point target can lie outside the two points.

E. Target Glint

Equation (19) gives the error signal out of the phase detector (with $K_\varepsilon = 1$) as

$$\varepsilon = \frac{\left(\Sigma_T + \Sigma_1^N \Sigma_{iI}\right)\left(\Delta_T + \Sigma_1^N \Delta_{iI}\right) + \left(\Sigma_1^N \Delta_{iQ}\right)\left(\Sigma_1^N \Sigma_{iQ}\right)}{|\Sigma + \Delta|\ |\Sigma|} \quad .$$

Until now the target signal has been assumed to originate from a single point source which produced the signals Σ_T and Δ_T. However, in actual practice the target consists of a number of separate scatterers whose return signals combine to form the Σ and Δ target signals and Σ_T and Δ_T do not actually exist. If only these separate scatterers are considered and all interference signals are ignored, Equation (19) will give the error signal with glint as

$$\varepsilon_T = \frac{\left(\Sigma_1^N \Sigma_{iI}\right)\left(\Sigma_1^N \Delta_{iI}\right) + \left(\Sigma_1^N \Delta_{iQ}\right)\left(\Sigma_1^N \Sigma_{iQ}\right)}{|\Sigma + \Delta|\ |\Sigma|}$$

where

$$|\Sigma| = \sqrt{\left(\Sigma_1^N \Sigma_{iI}\right)^2 + \left(\Sigma_1^N \Sigma_{iQ}\right)^2} \quad .$$

Since $\overline{\Sigma}_{iQ} = \overline{\Sigma}_{iI}$ and $\overline{\Delta}_{iQ} = \overline{\Delta}_{iI}$ where the bar denotes average value,

$$|\Sigma| = \sqrt{2}\, \Sigma_1^N \Sigma_i \quad .$$

For small angular errors $\Sigma_i \gg \Delta_i$; therefore,

$$|\Sigma + \Delta| \approx |\Sigma| \quad .$$

The complex target error signal then becomes

$$\varepsilon_T = \frac{2\left(\Sigma_1^N \Sigma_i\right)\left(\Sigma_1^N \Delta_i\right)}{2\left(\Sigma_1^N \Sigma_i\right)^2}$$

or

$$\varepsilon_T = \frac{\Sigma_1^N \Delta_i}{\Sigma_1^N \Sigma_i} \quad .$$

But $\sqrt{2}\, \Sigma_1^N \Sigma_i$ is the voltage in the Σ channel which has been designated as Σ_T. Thus

$$\varepsilon_T = \frac{\sqrt{2}\, \Sigma_1^N \Delta_i}{\Sigma_T} \quad .$$

But

$$\Delta_i = \frac{k_m}{BW} \theta_i \Sigma_i$$

where θ_i is the angle that produces voltages Δ_i and Σ_i, and may be defined as

$$\theta_i = \frac{\ell_i}{R} \qquad -\frac{L}{2} \le \ell_i \le \frac{L}{2}$$

where R is the range to the target and L is the length of the target in the plane of interest. Then ε_T becomes

$$\varepsilon_T = \frac{1}{R} \frac{\sqrt{2}}{\Sigma_T} \frac{k_m}{BW} \Sigma_1^N \ell_i \Sigma_i \quad . \tag{63}$$

If it is assumed that there are small rotations of the target about its center point, the target aspect angle will fluctuate and the reflector located at ℓ_i will reflect its radar energy with a Doppler shift of frequency of

$$f_i = \frac{2\, V_{ri}}{\lambda}$$

relative to the frequency of the signal from the target center. Vr_i is the relative velocity between the center of the target and the point located at ℓ_i. This relative velocity may be calculated from

$$V_{ri} = \omega_a \, \ell_i$$

where ω_a is the angular rotation of the target in rad/sec. Then

$$f_i = \frac{2\, \omega_a \, \ell_i}{\lambda} \quad . \tag{64}$$

If L is the length of the target in the plane of interest, then the maximum Doppler frequency variation from the target is

$$f_M = \frac{\omega_a \, L}{\lambda} \quad . \tag{65}$$

In actual practice, ω_a is not constant but varies with time because of such things as target vibration and internal motion. This can cause frequencies which are larger than f_M. Because of this, a better fit to the actual target spectrum is obtained with a Markoffian spectrum as shown in Figure 6 whose half power frequency is equal to f_M. The Markoffian spectrum may be generated by passing Gaussian noise having an amplitude of "A" through a single pole low pass filter whose transfer function is

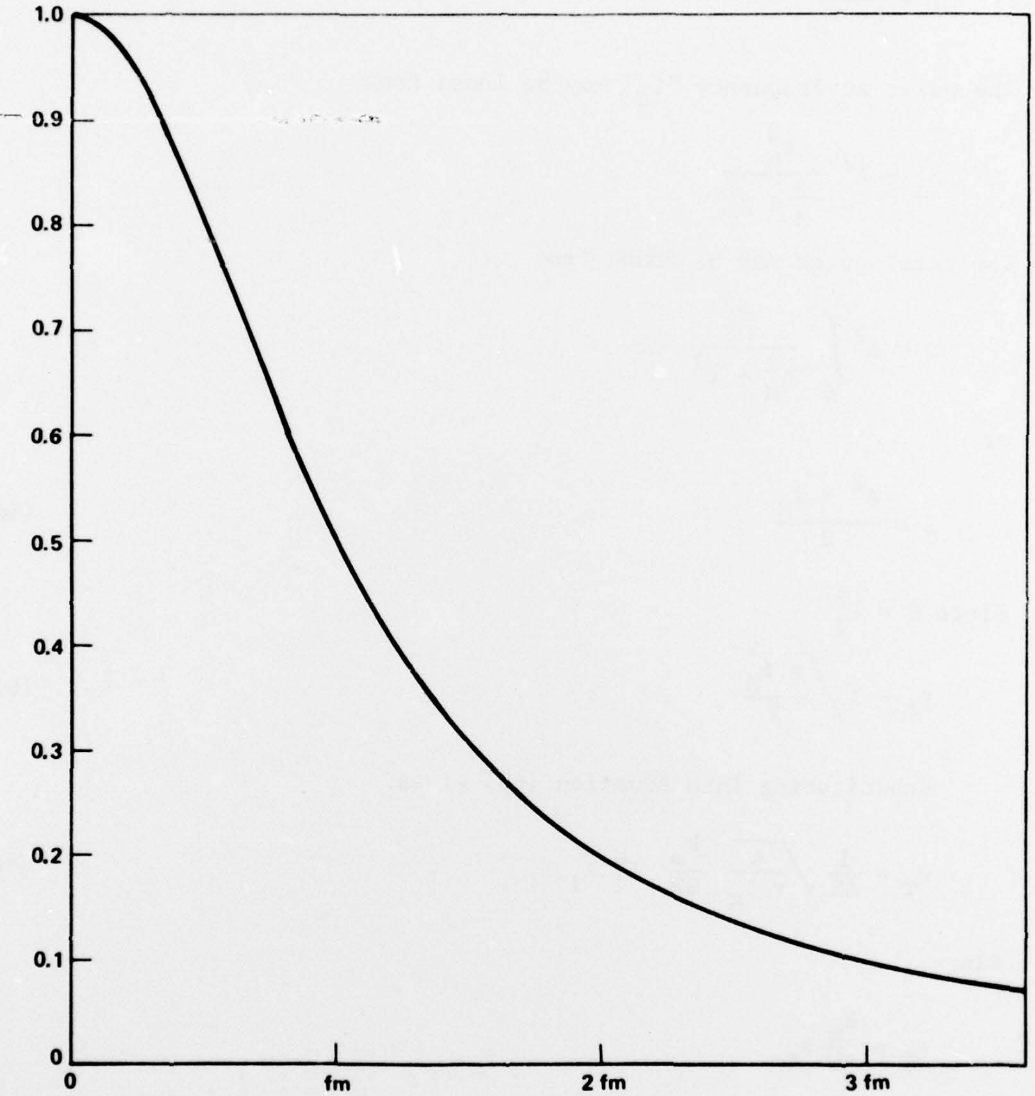

Figure 6. Markoffian power spectrum (normalized).

$$\frac{1}{1 + \frac{S}{2\pi\ f_M}} \quad .$$

The power at frequency "f_i" may be found from

$$S_i = A^2 \frac{f_M^2}{f_M^2 + f_i^2} \quad .$$

The total power may be found from

$$S = A^2 \int_0^\infty \frac{f_M^2}{f_M^2 + f^2}\ df$$

or

$$S = \frac{A^2\ \pi\ f_M}{2} \quad . \tag{66}$$

Since $S = \Sigma_T^2$

$$\Sigma_T = A \sqrt{\frac{\pi\ f_M}{2}} \quad . \tag{67}$$

Substituting into Equation (63) gives

$$\epsilon_T = \frac{1}{AR} \sqrt{\frac{4}{\pi\ f_M}}\ \frac{k_m}{BW}\ \Sigma_1^N\ \ell_i\ \Sigma_i \quad . \tag{68}$$

Since

$$\epsilon_T = \frac{k_m}{BW}\ \theta_T$$

where θ_T is the angle that the composite signal appears off boresight. Solving for θ_T results in

$$\theta_T = \frac{1}{AR} \sqrt{\frac{4}{\pi\ f_M}}\ \Sigma_1^N\ \ell_i\ \Sigma_i \quad . \tag{69}$$

The average value of θ_T is 0. A particular sample of θ is

$$\theta_G = \frac{1}{AR} \sqrt{\frac{4}{\pi f_M}}\, \ell_G\, \Sigma_G$$

with

$$\Sigma_G = A \frac{1}{1 + \tau S} \tag{70}$$

$$\tau = \frac{1}{2\pi f_M} \tag{71}$$

$$\ell_G = k_G\, \sigma_\ell \quad .$$

k_G is a Gaussian random number with a mean of 0 and a 1 σ value of 1. σ_ℓ is the 1 σ value of ℓ and typically varies from 1/6 to 1/3 of the maximum span of the target. Then

$$\theta_G = \sigma_\ell \frac{1}{R} \sqrt{\frac{4}{\pi f_M}}\, k_G \frac{1}{1 + \tau S} \quad . \tag{72}$$

Equation (72) may be expressed in block diagram form as shown in Figure 7.

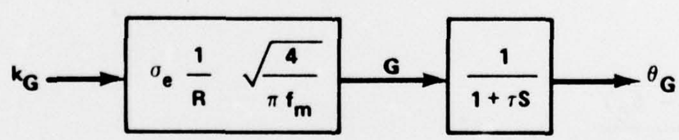

Figure 7. Glint model block diagram.

From Figure 7 it can be seen that

$$\dot{\theta}_G = \frac{G - \theta_G}{\tau} \tag{73}$$

where θ_G is the previous value of glint. $\dot{\theta}_G$ is the derivative of the present value of glint. The present value of glint may be obtained by utilizing the increment form of the Taylor series expansion expressed as

$$f(x + h) = f(x) + hf'(x) + \frac{h^2}{2!} f''(x) + \frac{h^3}{3!} f'''(x) + \ldots \tag{74}$$

where

$f(x + h) =$ New value of θ_G $(\theta_{G_{New}})$

$f(x) =$ Old value of θ_G

$h = \Delta t$ (i.e., time difference between samples).

Thus

$$\theta_{G_{New}} = \theta_G + \Delta t\, \dot{\theta}_G + \frac{\Delta t^2}{2!} \ddot{\theta}_G + \frac{\Delta t^3}{3!} \dddot{\theta}_G + \ldots \tag{75}$$

where $\dot{\theta}_G$ is defined by Equation (73) and

$$\ddot{\theta}_G = \frac{(\dot{G} - \dot{\theta}_G)}{\tau}$$

and

$$\dddot{\theta}_G = \frac{(\ddot{G} - \ddot{\theta}_G)}{\tau} .$$

Rearranging

$$\ddot{\theta}_G = \frac{\dot{G}}{\tau} - \frac{G}{\tau^2} + \frac{\theta_G}{\tau^2} \tag{76}$$

$$\dddot{\theta}_G = \frac{\ddot{G}}{\tau} - \frac{\dot{G}}{\tau^2} + \frac{G}{\tau^3} - \frac{\theta_G}{\tau^3} . \tag{77}$$

Substituting into Equation (75)

$$\theta_{G_{New}} = \theta_G + \Delta t \left(\frac{G - \theta_G}{\tau}\right) + \frac{\Delta t^2}{2!}\left(\frac{\dot{G}}{\tau} - \frac{G}{\tau^2} + \frac{\theta_G}{\tau^2}\right)$$

$$+ \frac{\Delta t^3}{3!}\left(\frac{\ddot{G}}{\tau} - \frac{\dot{G}}{\tau^2} + \frac{G}{\tau^3} - \frac{\theta_G}{\tau^3}\right) + \ldots \quad .$$

Rearranging terms

$$\theta_{G_{New}} = \left(1 - \frac{\Delta t}{\tau} + \frac{\Delta t^2}{2!\tau^2}\right)\theta_G + \left(\frac{\Delta t}{\tau} - \frac{\Delta t^2}{2!\tau^2} + \frac{\Delta t^3}{3!\tau^3}\right) G$$

$$+ \left(\frac{\Delta t^2}{2!\tau} - \frac{\Delta t^3}{3!\tau^2}\right)\dot{G} + \frac{\Delta t^3}{3!\tau}\ddot{G} + \ldots \quad .$$

Defining

$$A = 1 - \frac{\Delta t}{\tau} + \frac{1}{2!}\left(\frac{\Delta t}{\tau}\right)^2 + \ldots = e^{-\left(\frac{\Delta t}{\tau}\right)}$$

$$B = 1 - A$$

$$C = \Delta t - \tau B$$

gives

$$\theta_{G_{New}} = A\,\theta_G + B\,G + C\,\dot{G} + \ldots \qquad (78)$$

with G and its derivatives defined by

$$G = k_G\,\sigma_\ell\,\frac{1}{R}\sqrt{\frac{4}{\pi\,f_M}} \quad . \qquad (79)$$

This value of $\theta_{G_{New}}$ may now be considered as one of the error angles to be included in Equation (22).

IV. ANTENNA POSITION

The outputs of the phase detectors have been shown in Equations (10) and (11) to be of the form

$$\varepsilon = K_{\psi} \sin \left(\frac{\pi d}{\lambda} \sin \alpha\right) \tag{80}$$

where sin α is defined by either Equation (12) or Equation (13) depending upon the plane of interest. If sin α is small, then ε may be approximated by

$$\varepsilon = K \left(\psi_B - \psi_A\right) \tag{81}$$

where ψ_B is the target angle and ψ_A is the antenna angle. If K_{ψ} is equal to $\lambda/\pi d$, the gain constant "K" is 1.

A block diagram of the antenna tracking loop reflecting Equation (81) is shown in Figure 8.

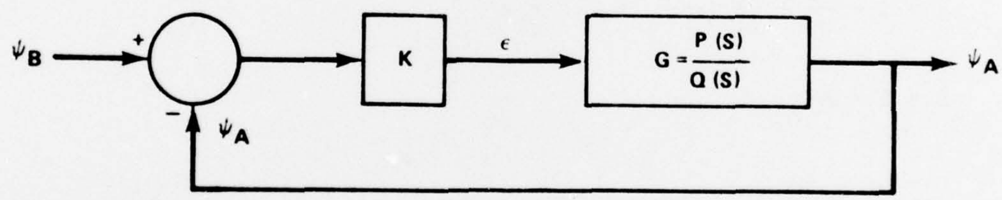

Figure 8. Antenna tracking loop.

Since $\psi_A = G\varepsilon$,

$$\varepsilon = \frac{K}{1 + KG} \psi_B \quad . \tag{82}$$

With

$$G = \frac{P(S)}{Q(S)} = \frac{b_m S^m + b_{m-1} S^{m-1} + \ldots + b_0}{a_n S^n + a_{n-1} S^{n-1} + \ldots + a_0} \tag{83}$$

the error signal becomes

$$\varepsilon = \frac{K[a_n S^n + a_{n-1} S^{n-1} + \ldots + a_0]}{[a_n S^n + a_{n-1} S^{n-1} + \ldots + a_0] + K[b_m S^m + b_{m-1} S^{m-1} + \ldots + b_0]} \psi_B \quad . (84)$$

It is required that $\psi_A \to \psi_B$, i.e., that the steady state value of $\varepsilon \to 0$. If the target angle ψ_B changes at a constant rate (V), it may be expressed as

$$\psi_B = Vt + \psi_0$$

where ψ_0 is its initial position. In the S-domain

$$\psi_B = \frac{V}{S^2} + \frac{\psi_0}{S} \quad .$$

Applying the final value theorem to Equation (84) gives the steady state value of ε as

$$\varepsilon_{SS} = \lim_{S \to 0} \left[S \frac{K(a_n S^n + a_{n-1} S^{n-1} + \ldots + a_1 S + a_0)}{(a_n S^n + a_{n-1} S^{n-1} + \ldots + a_1 S + a_0) + K(b_m S^m + b_{m-1} S^{m-1} + \ldots + b_0)} \left(\frac{V}{S^2} + \frac{\psi_0}{S} \right) \right]$$

or

$$\varepsilon_{SS} = \frac{a_1 V + a_0 \left(\frac{V}{S} + \psi_0 \right)}{\frac{a_0}{K} + b_0} \quad . \tag{85}$$

For ε_{SS} to be 0, a_1 and a_0 must be 0. This requires G from Equation (83) becomes

$$G = \frac{b_m S^m + b_{m-1} S^{m-1} + \ldots + b_1 S + b_0}{S^2 (a_n S^{n-2} + a_{n-1} S^{n-3} + \ldots + a_3 S + a_2)} \quad .$$

This transfer function consists of both the transfer function of the rate gyro which is 1/S and the transfer function of the controller G_c expressed as

$$G_c = \frac{b_m S^m + b_{m-1} S^{m-1} + \ldots + b_1 S + b_0}{S(a_n S^{n-2} + a_{n-1} S^{n-3} + \ldots + a_3 S + a_2)} .$$

A simple form of G_c to implement is

$$G_c = \frac{b_1 S + b_0}{a_2 S}$$

or

$$G_c = \frac{K_G\ (\tau S + 1)}{S} \tag{86}$$

where

$$K_G = \frac{b_0}{a_2} \tag{87}$$

$$\tau = \frac{b_1}{b_0} . \tag{88}$$

This is the familiar proportional plus integral controller. With this controller, the steady state error can be made to approach 0 if the angular rate of the target is constant.

With the proportional plus integral controller, it has been shown that a target that presents a constant angular rate to the radar can be tracked with an error approaching 0. If the angular rate is not constant but has a constant acceleration (A), the target angle may be expressed as

$$\psi_B = \frac{1}{2} At^2 + Vt + \psi_0 .$$

In the S-domain

$$\psi_B = \frac{A}{S^3} + \frac{V}{S^2} + \frac{\psi_0}{S} .$$

Applying the final value theorem to Equation (84) results in a steady state error of

$$\varepsilon_{SS} = \frac{a_2 A}{b_0} \quad .$$

Since a_1 and a_0 are both 0 with the proportional plus integral controller, substituting Equation (87) into the above results in

$$\varepsilon_{SS} = \frac{A}{K_G} \quad . \tag{89}$$

Thus with a proportional plus integral controller an angular acceleration produces a tracking error proportional to the acceleration.

V. SUMMARY

Expressions for the target signal voltages out of the phase detectors of a monopulse radar have been derived. Interference signals were then added to the target signal to determine the effects of the interference upon the phase detector output. The following expression was derived which expresses the 1 σ value of angular interference as a function of radar parameters and the signal-to-interference ratio.

$$\sigma_i = \frac{BW}{k_m \sqrt{2\left(\frac{S_\Sigma}{I_{\Delta i}}\right) n_{ei}}} \quad .$$

Interference terms were then obtained for thermal noise, rain, ground clutter, multipath, and target glint.

A brief analysis was performed on the radar antenna tracking loop. It was shown that a constant angular velocity target can be tracked with essentially no error due to the control loop if a proportional plus integral controller is utilized. However, if the angular velocity is not constant, a tracking error results which is proportional to the angular acceleration.

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